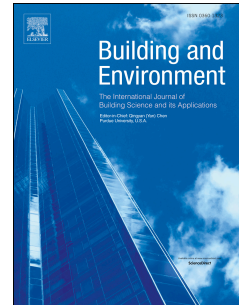


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Cascading temperature demand: the limits of thermal nesting in naturally ventilated buildings

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Abstract

A temperature cascade refers to a series of spaces that are progressively warmer or cooler than the exterior. Thermal nesting refers to architectural configurations that produce temperature cascades. This study shows how to design a thermally nested scheme that generates a natural temperature cascade, which occupants may adapt to with the change of seasons or during a heatwave. A steady-state model for any temperature cascade ventilated by thermal buoyancy is presented. The thermal comfort and energy-saving limits are analyzed compared to a single space of uniform temperature, ventilated by natural mixing. Adaptive comfort and heat stress theory inform the choice of stepwise temperatures in the cascade. The most comfortable space in the cascade has supplementary heating or cooling, but only when necessary. The buoyancy ventilation occurs separately in each space, or through each space in series. Heat recycling occurs through the partition walls. The results suggest that a high degree of thermal nesting can reduce energy demand by more than 50% in moderately hot or cold exterior conditions. The relative energy savings increase with the degree of nesting, the benchmark envelope conductance, and the size of the temperature increments. Three-step cascades save slightly more energy than two-step cascades, because the envelope losses are smaller. Flow-divided configurations save more energy than flow-connected ones, because the ventilation heat loss is divided into smaller flows with smaller temperature increments.

Keywords: thermal nesting, adaptive comfort, nudging, buoyancy ventilation, sustainable cooling, mixed-mode ventilation

Nomenclature

List of symbols

a_i	conductance distribution coefficient [-]
b_i	heating distribution coefficient [-]
c_i	ventilation distribution coefficient [-]
C_m	specific heat capacity of thermal mass [J/kg/°C]
C_p	specific heat capacity of air [kJ/kg/°C]
h	convective heat transfer coefficient [W/m ² /K]
H	active heating or cooling per person [W/person]
H_{int}	internal heat gains per person [W/person]
j	threshold UTCI value [-]

l	effective thickness of the thermal mass [m]
N	number of nested spaces [-]
PC	conductive heat transfer between spaces across partition i [W/person]
Q	airflow rate per person [$m^3/s/person$]
T	air temperature [$^{\circ}C$]
t	characteristic timescale [s]
UA	outside envelope conductance [W / K/person]
v_p	vapour pressure [kPa]
u	wind speed [m/s]
V	volume of the ventilated space [m^3]
x	substitute for T_{ext} [$^{\circ}C$]
α	thermal nesting factor [-]
β	heating distribution factor [-]
γ	ventilation distribution factor [-]
κ_m	thermal diffusivity of the thermal mass [m^2/s]
ρ	air density [kg/m^3]
ρ_m	thermal mass density [kg/m^3]
θ_i	air temperature in space i relative to the outside temperature [$^{\circ}C$]
ω	frequency of diurnal environment fluctuations [s^{-1}]
$\cap$	intersection point with [-]
Subscripts	
c, max	maximum comfortable interior
c, min	minimum comfortable interior
e	instantaneous exterior
ext	mean running exterior
f	floating
$free$	free-running
i	index of the characterized space ($i= 1, N$), or index of timescale (eg. t_1, t_2 , etc)
p	partition
ra	radiant
Acronyms	
ACM	Adaptive Comfort Model
IPCC	Intergovernmental Panel on Climate Change
UTCI	Universal Thermal Climate Index

1. Introduction

There are broadly two approaches to reducing the energy demand for heating and cooling in buildings. One is to focus on reducing envelope and ventilation heat losses (or gains). An alternative approach is to reduce temperature demand, which involves integrating adaptive measures into the design and operation of the building. The idea is to “nudge” behaviour, so occupants tolerate or desire some degree of natural thermal variation from season to season or room to room [1]. For instance, the 2021 Pritzker prize winners Lacaton and Vassal have developed a much-lauded retrofitting strategy focused on reducing temperature demand. Not only do they treat embodied carbon as an asset, vowing to “never demolish”, but they also design the interior spaces in a series of microclimates that are progressively warmer or cooler than the exterior [2]. The intermediate spaces are supposed to be “totally usable even if [they are] not the right temperature all [...] year [2].”



Figure 1: Examples of thermal nesting. **a.** Social housing retrofit, Cité du Grand Parc, in Bordeaux, France, by Lacaton and Vassal (2017). (Image used with permission of photographer). **b.** House in Takaya, Japan, by Suppose Design Office (2011). Note the sliding glass doors. (Image used with permission of author). **c.** Traditional courtyard in Cordoba, Spain, recently monitored in terms of adaptive comfort [12] (Creative Commons 4.0).

If this represents the start of a new trend in so-called sustainable architecture, there is undoubtedly a rich cultural history from which to draw. Just a small sample of architectural terms suggests that thermal thresholds have long been part of public and private life: arcade, atrium, antechamber, courtyard, cloister, doma (Japanese), enfilade, forstofa (Icelandic), foyer, lobby, loggia, qanitchaq (Iñupiat), sahn (Arabic), siinaq (Alutiiq), tuulikaappi (Finnish), vestibule. Consider also that interior plans used to be designed as a matrix of interconnected chambers, at least before the appearance of corridors in the late 15th century as a device for removing traffic from rooms [3]. It is therefore not hard to imagine historic buildings producing temperature cascades across numerous interconnected spaces.

In the present study, a temperature cascade refers to a *series* of progressively warmer or cooler spaces compared to the exterior. Meanwhile, thermal nesting refers to architectural configurations designed to produce temperature cascades. Figure 1 shows some contemporary and vernacular examples. Despite its widespread use, the energy-saving potential and adaptive limits of thermal nesting have not been investigated thoroughly.

Pitts & ben Saleh estimated the energy savings from relaxed temperature set points in transition spaces such as foyers, lobbies, atria, and ancillary spaces [4]. Relative energy savings were calculated when the setpoint temperature was offset by 5°C. Four building layouts were analyzed in a temperate UK climate. The layouts with large transition spaces around the perimeter yielded the largest relative savings: 33% in heating and 6% in cooling. Kansara monitored the effect of a 1°C transition zone setpoint increase in ten occupied air-conditioned towers in Abu Dhabi, UAE [5]. No clear association between temperature demand and energy use was observed. However, occupants' average comfort ratings in the transition zones improved in all but one of the buildings where the setpoint was adjusted. In Canada, a group of researchers examined the potential of thermal nesting as a strategy for various buildings, including historic retrofit. Their modelling results included mechanical heat recovery and indicated potential savings of up to 85% relative to a benchmark building [6]–[8].

Existing thermal comfort standards have been developed for indoor environments, while heat stress indicators have been developed for outdoor conditions. However, temperature cascades are a hybrid of interior and exterior conditions, and there is no specific guidance for designing the thermal conditions in a temperature cascade. Some researchers have nevertheless used Adaptive Comfort Models (ACM) to study thermal conditions in courtyards, lobbies, and atria [4], [9]–[14]. A recent study revealed how two vernacular courtyards in Cordoba, Spain, remained comfortable according to adaptive comfort standards for more than 75% of the time during summer [12]. Meanwhile, observations of how people adapt in transition spaces confirm some unsurprising patterns [10], [11]. Occupants tend to be more active in transition spaces than office settings where activities are more sedentary. They also tend to wear clothing more suited to exterior conditions. Moreover, they are more likely to adapt if they feel too hot or cold, adjusting their clothing level or moving to or from the shade [10]. Overall, the evidence suggests that occupants willingly accept a wide range of thermal conditions in transition spaces, if only for a short span of time.

There is a growing body of research on dynamic thermal perception, which refers to the psycho-physiological factors that influence occupants' perception of comfort and sensation in transient conditions, such as moving through a temperature cascade [13], [15]–[20]. Studies suggest occupants quickly become habituated or primed for thermal conditions. That is, people begin to expect a warmer-than-neutral experience after being exposed momentarily to slightly warm conditions and vice-versa for slightly cool conditions [18], [19]. Meanwhile, research on thermal alliesthesia considers the *affective* side of thermal experience [15], [16]. For instance, thermal sensations that are not comfortable may be experienced as pleasurable if they help restore the body to thermal equilibrium. Similarly, a gradual progression towards the neutral temperature positively affects how occupants perceive their final destination [13], [20]. However, questions remain regarding the mechanisms of dynamic thermal perception, how best to measure it, and how habituation to cold and heat discomfort differs [15].

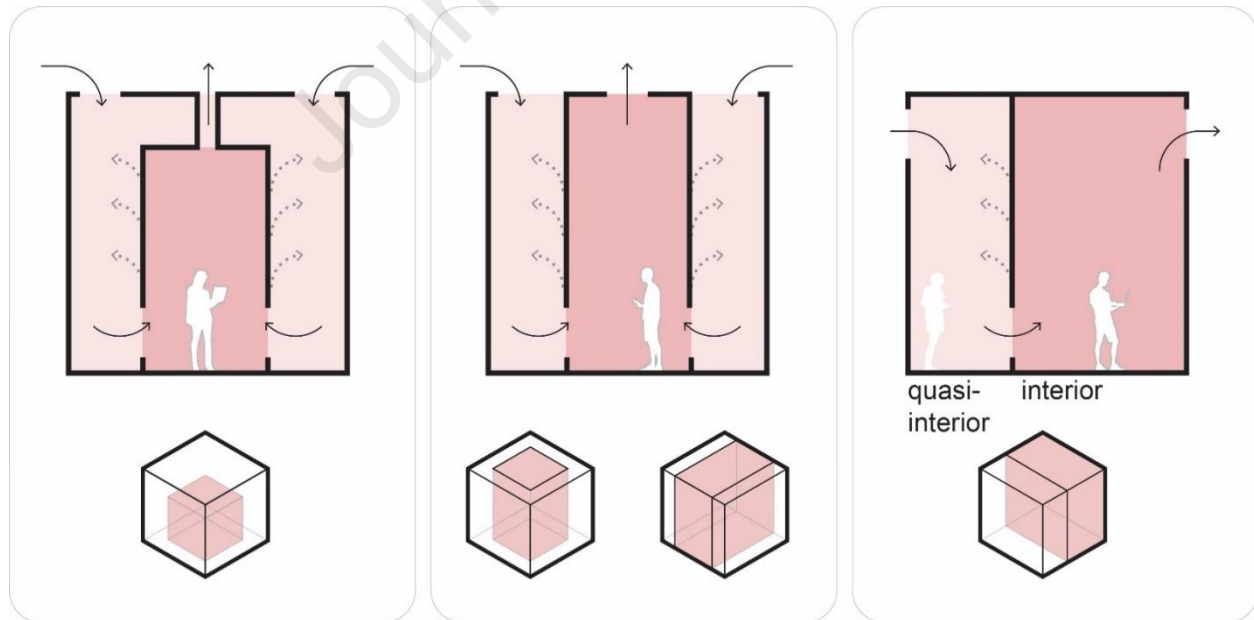


Figure 2: Different thermal nesting configurations with two thermal zones, from highly nested (left) to partially (right). Ventilation flow is driven by temperature differences between the quasi-interior and interior spaces, as explained in [23]. [2-column fitting image]

Another germane research topic is the interaction of temperature cascades and natural ventilation. The idea that a temperature cascade could power the ventilation flow is particularly appealing. Linden, Lane-Serff, and Smeed published what is now the canonical study on the fluid mechanics of buoyancy ventilation, otherwise known as the ‘emptying of filling boxes’ [21]. They showed how a single space may be ventilated by either mixing or displacement, depending on the location of vent openings and the strength and distribution of internal heating. More recently, Woods, Fitzgerald, and Livermore compared the heating demand for natural mixing and displacement in cool periods, showing how using a single high-level vent avoids uncomfortable drafts and stratification and the need to preheat incoming air [22]. Chenvidyakarn and Woods investigated the fluid mechanics of a two-step temperature cascade, where buoyancy ventilation flows through both spaces in series [23]. Each space was heated by an independent source so the ratio of heating could be adjusted. The flow direction was determined by the vertical dimension of vent openings and the relative magnitude of heating in each space. Stable flow developed in the direction of the temperature cascade, from the weakly heated space to the strongly heated one, depending on the relative magnitude of heating. All three of the above studies relied on physical laboratory-scale models to inform and verify their models [21]–[23].

2. Problem Statement

A steady-state model for any temperature cascade ventilated by thermal buoyancy is developed. The thermal comfort and energy-saving limits are analyzed compared to a single space of uniform temperature, ventilated by natural mixing. Adaptive comfort and heat stress theory inform the choice of stepwise temperatures in the cascade. The most comfortable space in the cascade has supplementary heating or cooling, but only when necessary. The buoyancy ventilation occurs separately in each space or through each space in series. Internal gains are defined by activity. Heat losses to the exterior occur via the exhaust and the exterior envelope. Heat recycling occurs through the partition walls.

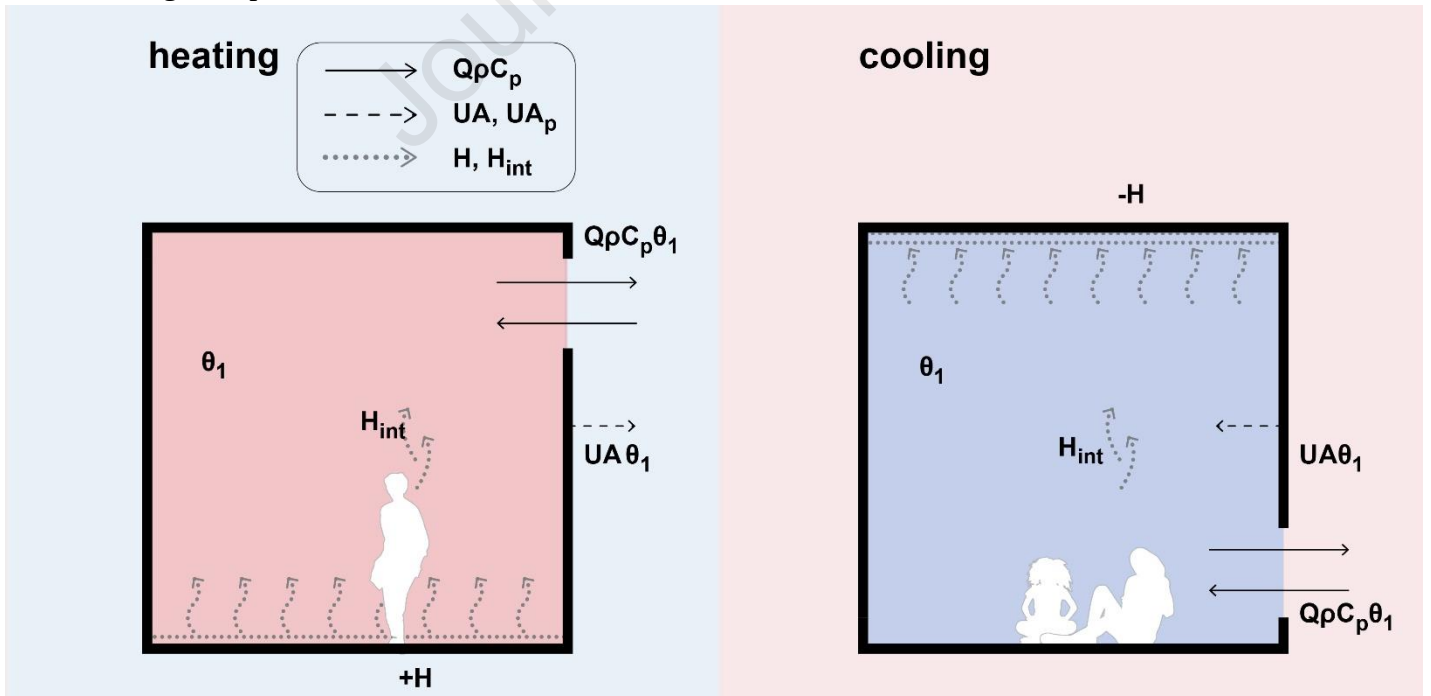


Figure 3: Schematic section of the single space reference building, ventilated through natural mixing in cooling (right) and heating (left). Warmer air temperature is indicated by red shading and cooler temperature is indicated by blue shading in this and subsequent figures. [2-column fitting image]

2.1 Reference case

Consider a building comprising a single space that is thermally uniform and ventilated via natural mixing, as presented schematically in Figure 3. This is the reference case. In heating mode (on the left), the air inside the space is warmer than outside, and fresh air enters and leaves the room through a high single vent, driven by thermal buoyancy. Heating is distributed along the floor to enhance convective mixing and comfort [24]. The plume of cool incoming air is preheated as it falls through the room, “mixing” with the warm interior air so that it reaches a comfortable temperature approaching the ambient by the time it reaches the occupied level [22], [25]. In cooling mode (on the right), the indoor air is cooler than the outside air, and fresh air enters and exits the building through a vent near the floor. Active cooling is distributed along the ceiling.

To provide a visual reference of the flow dynamics, Figure 4 on the right shows images of a salt bath scale model of a heated building vented through mixing ventilation. These stills are drawn from a video (available online) that compares mixing ventilation to the better-known upward displacement ventilation [26]. Woods et al. [22] evaluated the heating needs for natural mixing ventilation and found it to be energetically equivalent to displacement ventilation under most conditions, while it was more efficient for highly insulated buildings with high internal heat gains, as shown in Figure 4. Here, the single-space reference building is assumed to be vented through mixing ventilation.

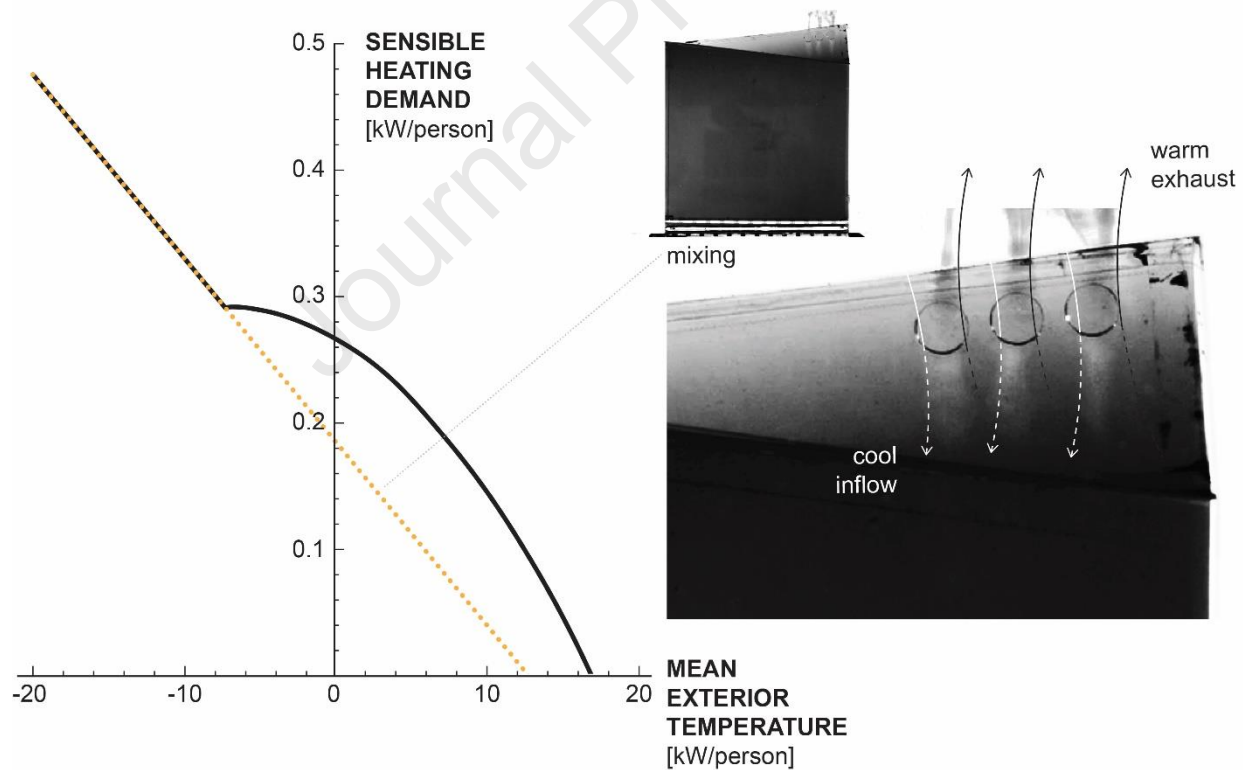


Figure 4. [Left]: comparison of heating needs in buoyancy-driven mixing (dotted) and upward displacement ventilation (solid), after analysis presented in [22]. The “bump” in the displacement ventilation is caused by the increased flow rate required to evacuate excess heat from internal heat gains. This additional airflow must be actively preheated before entering the space at occupant level to avoid cold drafts. (The higher load scenario from [22] is assumed.) Mixing ventilation is more energy efficient because no active pre-heating is required. Instead, it happens naturally: incoming fresh air mixes spontaneously with the outgoing air and with the interior. [Right]: full (top) and detail views (bottom) of a salt bath model showing mixing ventilation. [2-column fitting image]

It is also assumed that the ventilation combines with the heating or cooling to produce steady, well-mixed conditions with uniform temperature T_1 [°C]. Past research provides insight into factors that promote mixing in simple configurations [24], [27]. However, more research is needed to understand mixing in other scenarios, for instance, a heated wall instead of a heated floor [28]–[30].

2.2 Nested configurations

A building divided into two (Figure 5, top) or three (Figure 5, bottom) spaces is now considered, with the spaces maintained along a two- or three-step cascade of temperatures. The innermost, most deeply nested “interior” space (labeled 1 in Figure 5) is maintained at a setpoint temperature T_1 expected to be comfortable for most occupants. The “quasi-interior” space (labeled 2) is maintained at a more flexible and thus often less comfortable temperature T_2 , and the third “quasi-exterior” space that appears in the triple-space configuration (labeled 3) is maintained at an even more flexible setpoint temperature T_3 . In other words, the outer spaces are allowed to be cooler than the interior space in heating mode, and warmer in cooling mode.

The relative temperature θ_i of each space i can be described in terms of its setpoint temperature T_i relative to the instantaneous exterior temperature T_e :

$$\theta_i = T_i - T_e \quad 1$$

These cascading temperatures can then be described mathematically as $|\theta_1| > |\theta_2| > |\theta_3|$, with $\theta_i > 0$ in heating mode (on the left in Figure 5) and $\theta_i < 0$ in cooling mode (on the right).

Like the reference scenario, the nested buildings are naturally ventilated, but the overall flow structure depends on the configuration of the internal partition(s) that separate(s) the spaces. In the *flow-divided* mode shown in Figure 5, the partitions are continuous, preventing any air exchange between spaces. Each space is therefore vented independently, directly to the outside. In this mode, the flow structure in each space is the same as in the single space reference case. As in the reference, vents to the outside are opened near the ceiling in heating mode and near the floor in cooling mode.

In the *flow-connected* mode shown in Figure 6, each partition is equipped with vents such that a single, shared ventilation stream flows through the whole building. Fresh outside air flows from the most-exterior to the most-interior space through the partition vents, until it exhausts back to the outside from space 1. The fresh air is naturally preheated as it passes through the more exterior space(s). As in the reference and flow-divided cases, exhaust vents are opened near the ceiling in heating mode and near the floor in cooling mode. Here, however, air is intended to flow through each vent in only one direction. The vent elevation in each partition alternates to facilitate mixing and enhance thermal comfort.

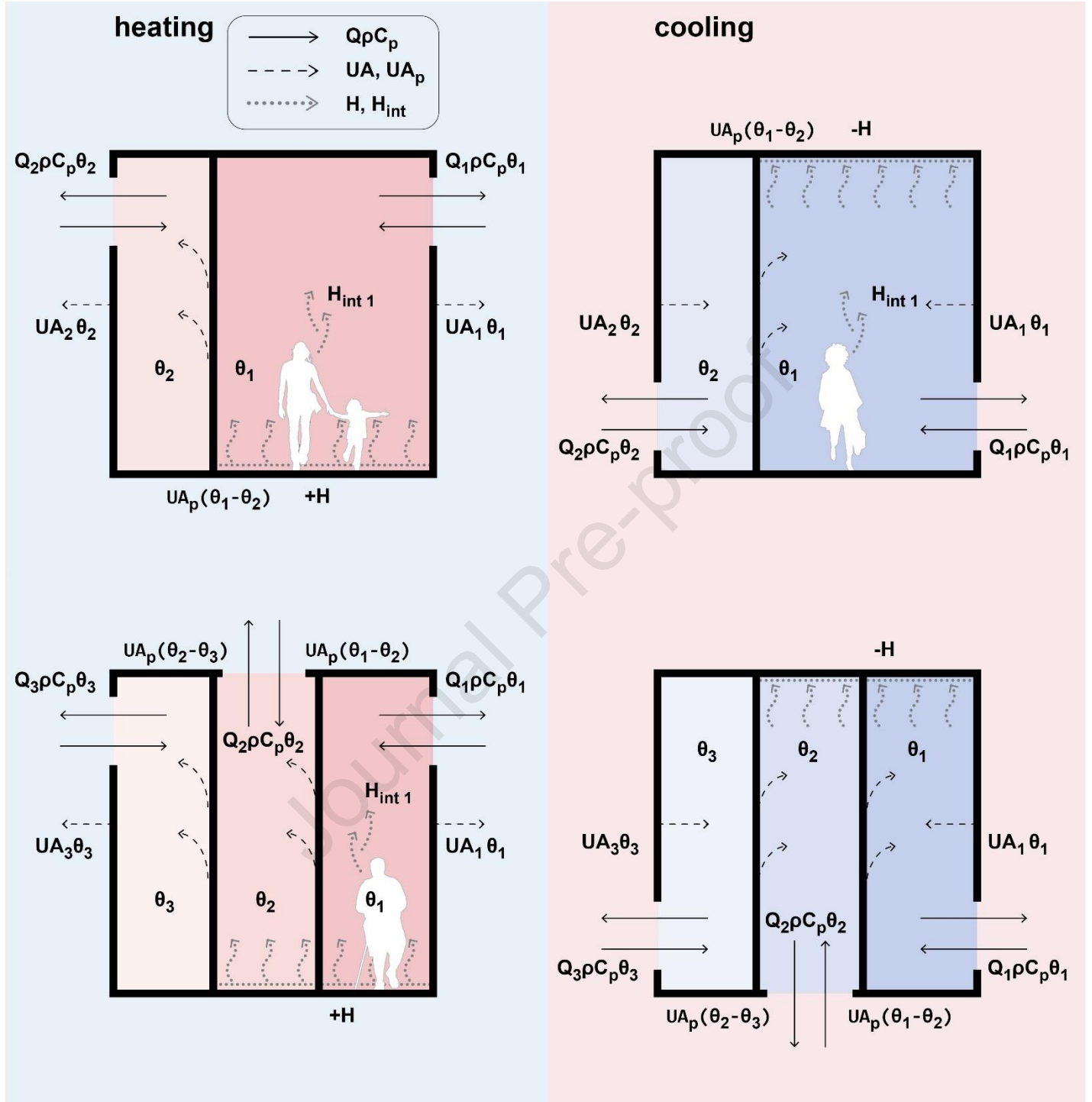


Figure 5: Section views of flow-divided two-step (top) and three-step (bottom) nested configurations in heating (left) and cooling (right). Each space i has its own conductance (UA_i), ventilation rate (Q_i), internal heat gains ($H_{int i}$), and active conditioning (H) required to maintain the relative setpoint temperature θ_i . Each space is vented independently, directly to the outside. [2-column fitting image]

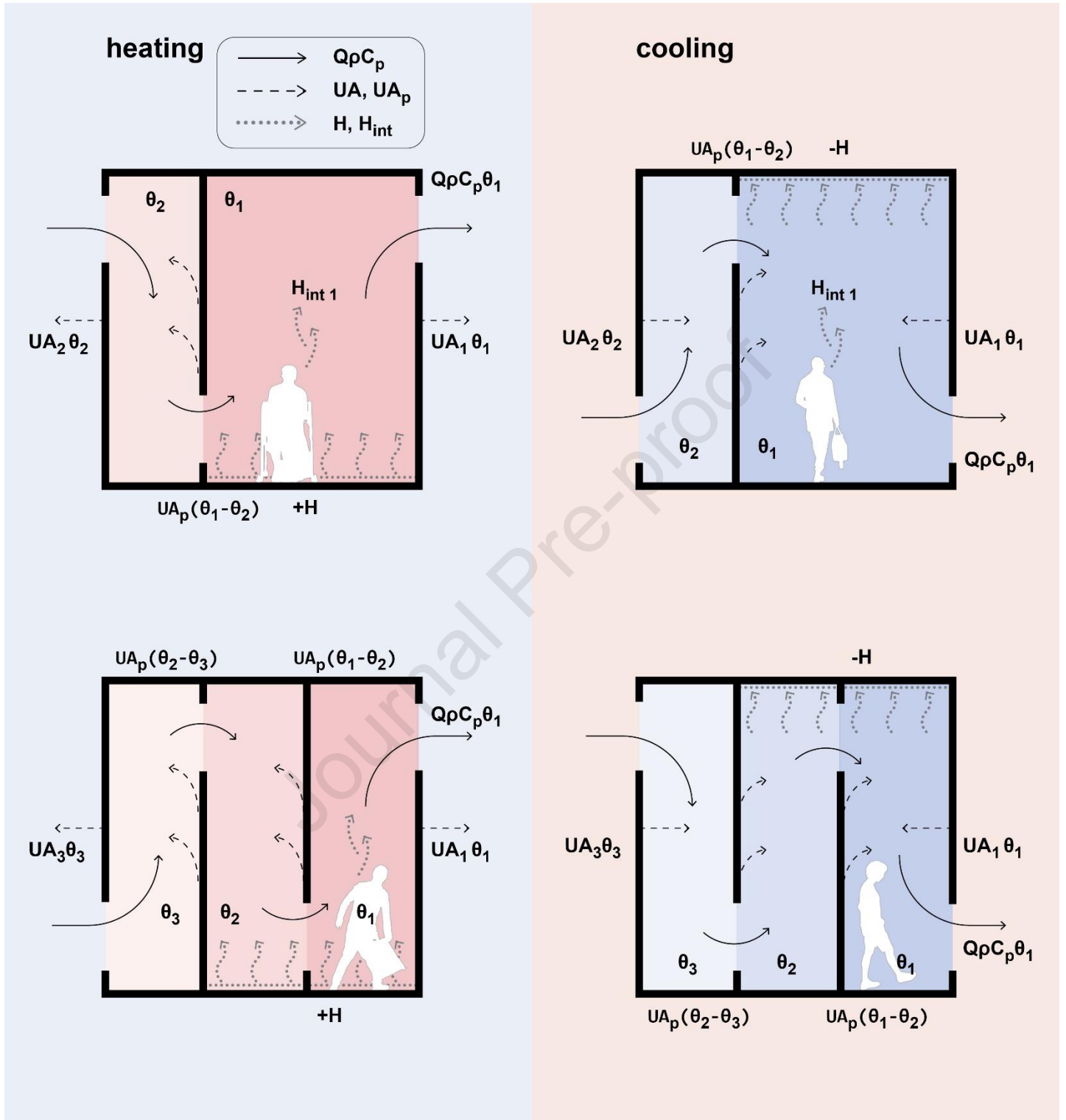


Figure 6: Section views of flow-connected two-step (top) and three-step (bottom) thermal nesting, in heating (left) and cooling mode (right). Parameters are defined in Figure 5. [2-column fitting image]

To ensure conditions are thermally well-mixed in each space, dominant heating must be uniformly distributed across the base of the space [23], [24]. The presence of heat sources that are localized [31] or distributed along a vertical surface [29], [30], [32] tend to generate vertical stratification. The mechanics of buoyancy are such that the processes and trends may be applied to cooling mode as well by simply inverting the top and bottom (e.g., [33]), as shown on the right in Figures 3, 5 and 6. While the understanding of buoyancy driven ventilation flows continues to evolve, existing work [23] provides a sufficiently solid base for the present model as a starting point. The pressure gradients that drive the flow are not discussed here, but more details can be found in Chenvidyakarn and Woods [23]. The analysis focuses instead on the energy balance.

3. Methodology

3.1 Energy balance

The equilibrium temperature of each space is determined by the balances of internal heat sources/sinks and the heat exchanged with neighboring spaces or the exterior environment. For a single-space building, this energy balance may be written as

$$H + H_{int} = Q\rho C_p \theta_1 + UA\theta_1 \quad 2$$

where H [W/person] is the amount of heat actively removed or added, H_{int} [W/person] are the internal heat gains due to occupants, equipment, etc., Q is the volumetric airflow rate [$m^3/s/person$], and UA is the conductance of the building envelope [W/K/person] [22]. Engineering standards for indoor thermal environment and air quality prescribe a range of air change rates, per-person or per-floor area ventilation rates, based on occupancy, building type, what percentage of dissatisfied occupants is tolerated, or on the limitation of specific contaminants. Prescribed per-person rates typically range from 4 to 10 liters per second [34]. Q is set here to the upper limit of this range ($0.01 m^3/s/person$), which is specified by ISO 17772 for a high level of indoor environmental quality [35]. The air density ρ is set to $1225 kg/m^3$, and its specific heat capacity C_p is set to $1.005 kJ/kg/^\circ C$. θ_1 is the excess of the interior setpoint temperature relative to the exterior temperature, $T_i - T_e$, as given by Eq. **Error! Reference source not found.**

On the right-hand side of Eq. 2, the first term $Q\rho C_p \theta_1$ represents the ventilation losses/gains, and the second term $UA\theta_1$ represents losses/gains via the building envelope. In heating mode, both θ_1 and thus the ventilation and envelope terms are positive, as warmer indoor temperatures relative to the exterior drive heat losses towards the outside. In cooling mode, these terms are negative, signifying heat gains as warmer outdoor temperatures drive heat transfer towards the cooler interior. Following the same pattern as θ_1 , the active heating/cooling term H is positive in heating mode when heat input is required and negative in cooling mode when heat must be removed. The absolute value of H is plotted in the results section, so both heating and cooling needs appear as positive in the figures. The internal heat gains term H_{int} is always positive.

The new model for nested buildings is developed from an existing model for two flow-connected spaces developed by [23]. First, the [23] model is generalized to allow spaces 1, 2 and 3 to each have their own conductance UA_i , internal heat gains $H_{int i}$, active heating H_i , and, in flow-divided mode, flow rate Q_i . The subscript i takes the appropriate value 1, 2 or 3 to indicate which space

the parameter characterizes. The resulting equations describe the energy balance of the full building for flow-divided double-space or “two-step” buildings,

$$H_1 + H_2 + H_{int\ 1} + H_{int\ 2} = Q_1 \rho C_p \theta_1 + Q_2 \rho C_p \theta_2 + UA_1 \theta_1 + UA_2 \theta_2 \quad 3$$

for flow-connected two-step buildings,

$$H_1 + H_2 + H_{int\ 1} + H_{int\ 2} = Q \rho C_p \theta_1 + UA_1 \theta_1 + UA_2 \theta_2 \quad 4$$

for flow-divided triple-space or “three-step” buildings,

$$\begin{aligned} H_1 + H_2 + H_3 + H_{int\ 1} + H_{int\ 2} + H_{int\ 3} \\ = UA_1 \theta_1 + UA_2 \theta_2 + UA_3 \theta_3 + Q_1 \rho C_p \theta_1 + Q_2 \rho C_p \theta_2 + Q_3 \rho C_p \theta_3 \end{aligned} \quad 5$$

and for flow-connected three-step buildings

$$H_1 + H_2 + H_3 + H_{int\ 1} + H_{int\ 2} + H_{int\ 3} = UA_1 \theta_1 + UA_2 \theta_2 + UA_3 \theta_3 + Q \rho C_p \theta_1. \quad 6$$

Note how a “local” envelope, active heating and internal heat gain term ($UA_i, H_i, H_{int\ i}$) now appears for each space. Local ventilation terms ($Q_i \rho C_p \theta_i$) appear in the equations in flow-divided mode but not in flow-connected mode, since it has only one ventilation stream and thus only one loss term $Q \rho C_p \theta_1$.

The energy balance for nested buildings with N spaces may be generalized using summation notation, which yields the following for flow-divided mode

$$\sum_{i=1}^N (H_i + H_{int\ i}) = \sum_{i=1}^N (Q_i \rho C_p \theta_i + UA_i \theta_i) \quad 7$$

and flow-connected mode

$$\sum_{i=1}^N (H_i + H_{int\ i}) = Q \rho C_p \theta_1 + \sum_{i=1}^N UA_i \theta_i. \quad 8$$

$N=2$ for two-step buildings and $N=3$ for three-step buildings.

3.2 Temperature cascade equations

The temperature cascade $\theta_1, \theta_2, \dots, \theta_N$ across the nested spaces is a core feature of thermal nesting. An expression for the relative temperature θ_i for each space is obtained by writing the local energy balance for each space i . For two-step flow-divided buildings, the resulting temperature equations for each space are

$$\theta_1 = \frac{H_1 + H_{int\ 1} - UA_1 \theta_1}{Q_1 \rho C_p} - \frac{UA_p (\theta_1 - \theta_2)}{Q_1 \rho C_p} \quad 9$$

$$\theta_2 = \frac{H_2 + H_{int\ 2} - UA_2 \theta_2}{Q_2 \rho C_p} + \frac{UA_p (\theta_1 - \theta_2)}{Q_2 \rho C_p} \quad 10$$

and for the three-step flow-divided mode they are

$$\theta_1 = \frac{H_1 + H_{int\ 1} - UA_1\theta_1}{Q_1\rho C_p} - \frac{UA_p(\theta_1 - \theta_2)}{Q_1\rho C_p} \quad 11$$

$$\theta_2 = \frac{H_2 + H_{int\ 2} - UA_2\theta_2}{Q_2\rho C_p} + \frac{UA_p(\theta_1 - \theta_2) - UA_p(\theta_2 - \theta_3)}{Q_2\rho C_p} \quad 12$$

$$\theta_3 = \frac{H_3 + H_{int\ 3} - UA_3\theta_3}{Q_3\rho C_p} + \frac{UA_p(\theta_2 - \theta_3)}{Q_3\rho C_p} \quad 13$$

The new terms $UA_p(\theta_1 - \theta_2)$ and $UA_p(\theta_2 - \theta_3)$ on the right designate the conductive heat transfer through the partitions separating each pair of adjoining spaces, where UA_p is the conductance of the partition. The sign in front of these terms switches between successive equations because heat is transferred from more interior to more exterior spaces in heating mode (when $\theta_i > \theta_{i+1}$) and vice versa in cooling mode (when $\theta_i < \theta_{i+1}$). In the three-step configuration, space 2 has two such terms with opposite signs since it gains heat from the cooler space adjoining it to one side and loses heat to the warmer space to the other side.

The temperature equations for flow-connected two-step buildings are

$$\theta_1 = \frac{H_1 + H_{int\ 1} - UA\theta_1 + Q\rho C_p\theta_2}{Q\rho C_p} - \frac{UA_p(\theta_1 - \theta_2)}{Q\rho C_p} \quad 14$$

$$\theta_2 = \frac{H_2 + H_{int\ 2} - UA\theta_2}{Q\rho C_p} + \frac{UA_p(\theta_1 - \theta_2)}{Q\rho C_p} \quad 15$$

and for three-step flow-connected buildings they are:

$$\theta_1 = \frac{H_1 + H_{int\ 1} - UA_1\theta_1 + Q\rho C_p\theta_2}{Q\rho C_p} - \frac{UA_p(\theta_1 - \theta_2)}{Q\rho C_p} \quad 16$$

$$\theta_2 = \frac{H_2 + H_{int\ 2} - UA_2\theta_2 + Q\rho C_p\theta_3}{Q\rho C_p} + \frac{UA_p(\theta_1 - \theta_2) - UA_p(\theta_2 - \theta_3)}{Q\rho C_p} \quad 17$$

$$\theta_3 = \frac{H_3 + H_{int\ 3} - UA_3\theta_3}{Q\rho C_p} + \frac{UA_p(\theta_2 - \theta_3)}{Q\rho C_p} \quad 18$$

The flow-divided and flow-connected modes differ in how the ventilation terms appear in Eqs. 9-13 and 14-18. For flow-connected buildings, a new ventilation term $Q\rho C_p\theta_{i+1}$ appears in the numerator of the expression for space i that represents thermal energy transported by the airflow from the upstream space $i+1$. This new term appears in the equation for each space except the most exterior, as its ventilation stream enters at the outside temperature.

In general form, the temperature equations for flow-divided nested buildings become

$$\theta_i = \frac{H_i + H_{int\ i} - UA_i \theta_i + PC_i}{Q_i \rho C_p} \quad 19$$

and for the flow-connected mode they become:

$$\theta_i = \frac{H_i + H_{int\ i} - UA_i \theta_i + PC_i + Q \rho C_p \theta_{i-1}}{Q \rho C_p}, \quad 20$$

where i takes the values 1 to N and θ_0 can be understood to refer to the outside temperature so that it equals zero. N may theoretically be increased ad infinitum in flow-divided mode. In flow-connected mode, cumulative energy loss as the air flows through each vent would limit how many spaces there can be and still maintain the desired flow rate driven by buoyancy. PC_i designates conductive heat transfer between spaces across the partition. It takes the following form for both sets of equations:

$$\begin{aligned} PC_1 &= -UA_p(\theta_1 - \theta_2) \\ PC_i &= +UA_p(\theta_{i-1} - \theta_i) - UA_p(\theta_i - \theta_{i+1}), \text{ for } i=2, N-1. \\ PC_N &= +UA_p(\theta_{N-1} - \theta_N) \end{aligned} \quad 21$$

3.3 Design parameters

As mentioned above, the temperature of an individual space i is primarily determined by its envelope conductance UA_i , ventilation rate Q_i , and internal heat gains $H_{int\ i}$. However, to allow direct comparison, the total UA , Q and H_{int} must be the same.

$$\begin{aligned} \sum_{i=1}^N UA_i &= UA \\ \sum_{i=1}^N H_{int\ i} &= H_{int} \\ \sum_{i=1}^N Q_i &= Q \end{aligned} \quad 22$$

The parameters for an individual space i are related to the total using the non-dimensional coefficients a_i , b_i , and c_i . The parameter a_i describes the proportion of the total envelope conductance allocated to that space. The parameter b_i describes its proportion of the total internal heat gains. Finally, the parameter c_i describes its proportion of the total ventilation.

$$\begin{aligned} UA_i &= a_i UA \\ H_{int\ i} &= b_i H_{int} \\ Q_i &= c_i Q \end{aligned} \quad 23$$

For example, $UA_1 = a_1 UA$, $UA_2 = a_2 UA$, etc.

The sum of each set of coefficients must therefore be equal to 1:

$$\sum_{i=1}^N a_i = \sum_{i=1}^N b_i = \sum_{i=1}^N c_i = 1 \quad 24$$

These expressions may easily be substituted into the energy balances (Eqs. 2-6) and temperature equations (Eqs. 9-18) to generate equations in terms of total conductance, heat gain, and ventilation flow. For example, the energy balance for the two-step flow-divided case (Eq. 3) becomes:

$$H_1 + H_2 + H_{int\ 1} + H_{int\ 2} = c_1 Q \rho C_p \theta_1 + c_2 Q \rho C_p \theta_2 + a_1 U A \theta_1 + a_2 U A \theta_2 \quad 25$$

The general form of the energy balance equations therefore becomes

$$H = \sum_{i=1}^N |(a_i U A \theta_i - b_i H_{int\ i} + c_i Q \rho C_p \theta_i)| \quad 26$$

for flow-divided mode, and

$$H = \left| Q \rho C_p \theta_1 + \sum_{i=1}^N (a_i U A \theta_i - b_i H_{int\ i}) \right|, \quad 27$$

for flow-connected mode, where $N=2$ for two-step buildings and $N=3$ for three-step buildings. The total conditioning needs for all N spaces are included in the term $H = \sum_{i=1}^N |H_i|$.

This raises three questions. First, what happens if the proportion of envelope conductance assigned to the quasi-interior and quasi-exterior spaces is increased relative to the interior space? The “nesting factor” α is defined, which describes the distribution of envelope conductance. α is given by the sum of the coefficients a_i for the quasi-interior and quasi-exterior ($i \geq 2$) spaces:

$$\alpha = \sum_{i=2}^N a_i \quad 28$$

Buildings with a higher value of α are described as having a higher “degree of nesting”.

Second, how does the distribution of internal heat gains influence relative savings? The “heating distribution” β is defined to describe what proportion of the internal heat gains are allocated to the more exterior spaces: it is also given by the sum of their coefficients:

$$\beta = \sum_{i=2}^N b_i \quad 29$$

Third, what happens in flow-divided mode if the total ventilation flow rate is divided differently amongst spaces? The “ventilation distribution” γ is defined, which describes the proportion of the total ventilation flow allocated to the more-exterior spaces:

$$\gamma = \sum_{i=2}^N c_i \quad 30$$

3.4 Adaptive comfort thresholds

The temperature increments between the different spaces are the final elements needed to quantify the sensible heating and cooling demand. As discussed in Section 2, the interior space should be maintained within a “comfortable” range of temperatures. However, the quasi-interior and quasi-exterior spaces need not be *comfortable*, only *more comfortable* than the exterior. Ideally, they would not be actively heated or cooled for much of the year, maintained in a close-to-free-running thermal “cascade” through heat exchange with the more interior spaces [4].

A combination of the building’s free-running temperature [36], the European Standard Adaptive Comfort Model [37] and the Universal Thermal Climate Index [38] is used to inform a cascade of setpoint temperatures which accounts for heat stress, comfort, and energy savings.

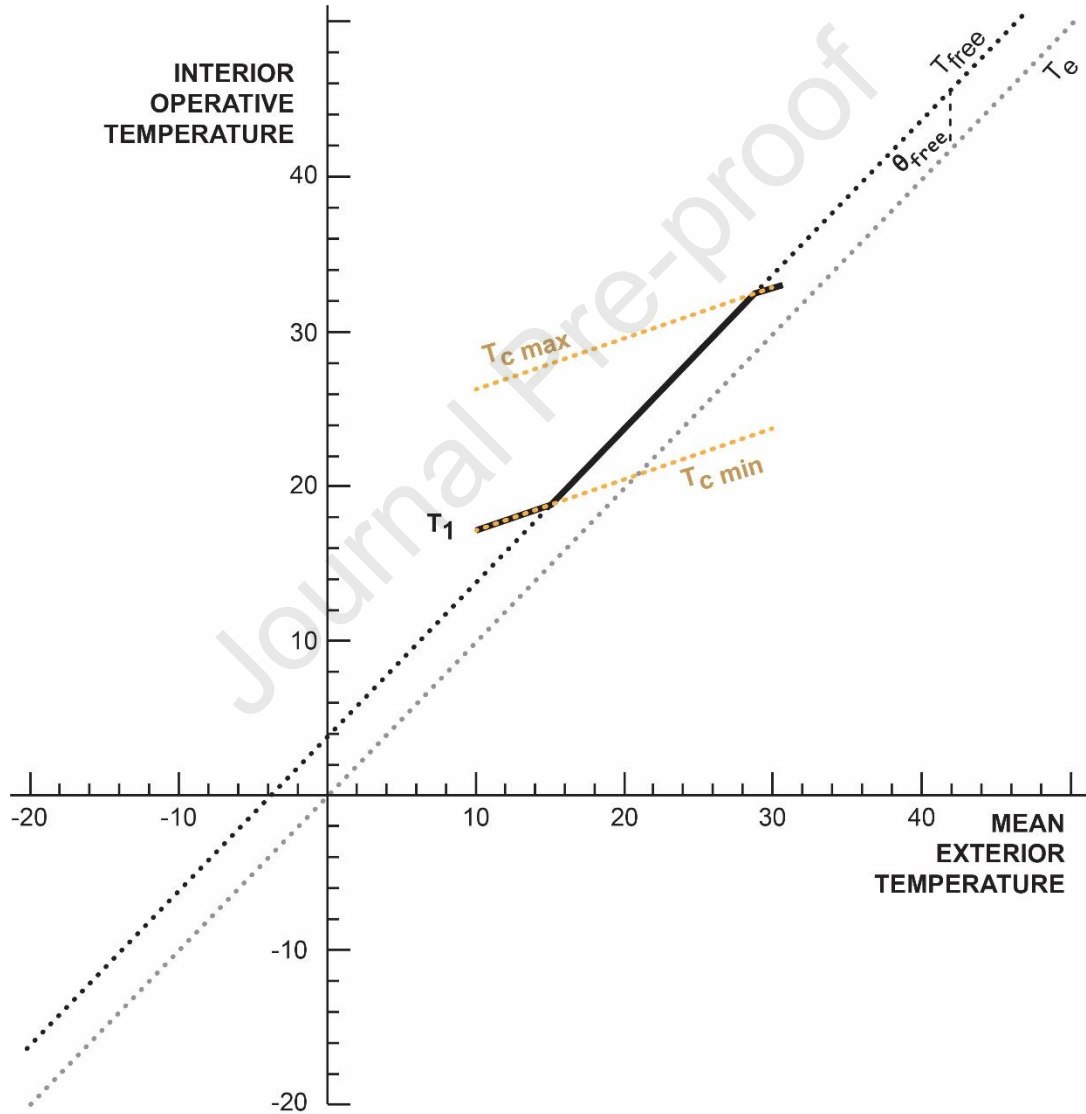


Figure 7: The setpoint temperature profile for the interior space T_1 (thick black line) as a function of the mean exterior temperature. T_e (thin dotted black line) is the instantaneous exterior temperature and T_{free} (thick dotted black line) is the building’s free-running temperature (Eq. 33). $T_{c\ max}$ and $T_{c\ min}$ (dotted gold lines) are the upper and lower limits of the adaptive comfort model [37].

Figure 7 shows the portion of the setpoint profile for T_1 (solid black line) where it is defined using the free-running temperature (thick dotted black line) and ACM (dotted gold lines). Following the ACM, the x -axis defines the “recent history” of the exterior temperature T_{ext} , taken as the outdoor running mean over the previous 10 to 30 days. The y -axis plots the interior operative temperature.

The ACM defines upper and lower limits for “comfortable” interior operative temperatures as a function of the prevailing exterior temperature:

$$T_{c\ max} = 18.8 + 0.33 T_{ext} + 4 \quad 31$$

$$T_{c\ min} = 18.8 + 0.33 T_{ext} - 5 \quad 32$$

where $T_{c\ max}$ is the upper limit and $T_{c\ min}$ the lower. The final term on the right-hand side determines the breadth of the “comfortable” band in the y -direction and is determined by the category of building in question. The values for Category III are used, which apply for “an acceptable, moderate level of expectation” [37].

The building’s free-running temperature T_{free} is the equilibrium temperature that the interior assumes in the absence of active heating and cooling [36]. When T_{free} falls within the limits defined by the ACM, it is considered comfortable and no active heating or cooling is needed.

The building must be cooled if its free-running temperature exceeds the ACM limits and heated if it falls below them. The interior should ideally be maintained at a comfortable temperature while minimizing energy use. Therefore, the T_1 profile is set to follow the free-running temperature as long as it falls within the ACM band. Beyond that, T_1 follows the upper limit of the ACM ($T_{c\ max}$) in cooling mode and its lower limit ($T_{c\ min}$) in heating mode.

The free-running temperature is offset above the instantaneous exterior temperature by some value θ_{free} :

$$T_{free} = T_{ext} + \theta_{free} \quad 33$$

For the purposes of this study, it is assumed that the prevailing exterior temperature T_{ext} is representative of the instantaneous exterior temperature. The offset θ_{free} is driven by the balance between internal heat gains and losses via the envelope and ventilation stream(s). For the single space reference building, θ_{free} is found by solving the energy balance (Eq. 2) with $H = 0$, yielding:

$$\theta_{free} = \frac{H_{int}}{UA + \rho C_p Q} \quad 34$$

Higher internal gains and/or lower conductances would cause a greater offset, which would simply shift the range where T_{free} intersects the ACM towards lower values of T_{ext} .

In the nested configurations, the internal gains, envelope conductance, and ventilation losses may differ for each space i , so the free-running temperature for each space $T_{free\ i}$ may also differ. Free-running temperatures specific to each space may be found by simultaneously solving the temperature equations (Eqs. 9-18) of the appropriate configuration for each relative temperature

θ_i , with all active conditioning terms H_i set to 0. For the sake of simplicity, analysis is restricted to the particular scenario where the internal heat gains H_{int} are distributed such that the free-running temperatures of each space are equal. This “balanced” scenario occurs when the internal heat distribution coefficients b_1 , b_2 , and b_3 respect the following constraints.

For two-step configurations in flow-divided mode, b_2 must equal

$$b_2 = \frac{c_2 Q \rho C_p + a_2 UA}{Q \rho C_p + UA} \quad 35$$

and in flow-connected mode,

$$b_2 = \frac{Q \rho C_p + a_2 UA}{Q \rho C_p + UA}. \quad 36$$

For the three-step configurations in flow-divided mode, b_2 and b_3 must equal

$$b_2 = \frac{c_2 Q \rho C_p + a_2 UA}{Q \rho C_p + UA} \quad 37$$

$$b_3 = \frac{c_3 Q \rho C_p + a_3 UA}{Q \rho C_p + UA}$$

and in flow-connected mode

$$b_2 = \frac{a_2 UA}{Q \rho C_p + UA} \quad 38$$

$$b_3 = \frac{Q \rho C_p + a_3 UA}{Q \rho C_p + UA}$$

These relations are obtained by simultaneously solving the relevant temperature equations for b_2 and b_3 while setting $\theta_1 = \theta_2 = \theta_3$ and $H_1 = H_2 = H_3 = 0$. In this “balanced” scenario, the free-running temperature of each space is equal to that of the reference case, $T_{free} = T_{ext} + \frac{H_{act}}{UA + \rho C_p Q}$.

This value can thus be used for all spaces and configurations when internal heat gains are balanced, simplifying analysis and direct comparison.

The ACM is limited to defining interior setpoint temperatures when the prevailing exterior temperature falls in the range $10^\circ\text{C} < T_{ext} < 30^\circ\text{C}$. Outside that range, the EN standard suggests fixed interior temperature setpoints around 19°C for heating and 27°C for cooling (these setpoints vary with the intended program).

3.5 Heat stress thresholds

The ACM is defined for interior spaces, but no guidance is provided for spaces such as the quasi-interior and quasi-exterior that need not be *comfortable* but should still be better than the exterior conditions. For more extreme exterior temperatures, the simplified Universal Thermal Climate

Index (UTCI) is therefore integrated with the ACM to create a continuous index between comfort and safety.

The UTCI is a biometeorology parameter [38] that uses a theoretical heat-budget model to describe thermal perception. It defines ten thermal stress categories, as shown in Table 1. One set of outputs from the model has been summarized as a polynomial equation which is a function of the mean radiant temperature (T_{ra} [$^{\circ}\text{C}$]), instantaneous air temperature (T_e [$^{\circ}\text{C}$]), wind speed (u [m/s]) and vapor pressure (v_p [kPa]). This simplified form of the UTCI assumes certain inputs to the heat budget which may not align with those made by the ACM, as is further discussed below. The clothing level [clo] is defined as a non-linear function of air temperature, meaning it is assumed that all occupants adjust their clothing to the same degree in response to the prevailing exterior temperature. The metabolic rate is set to 2.3 *MET*, corresponding to a constant 4 *km/h* walking speed. This is equivalent to roughly 133.4 *W/person*, which will be used later to set the internal heat gains (H_{int}) in the calculations.

UTCI < -40	extreme cold stress
-40 < UTCI < -27	very strong cold stress
-28 < UTCI < -13	strong cold stress
-13 < UTCI < 0	moderate cold stress
0 < UTCI < 9	slight cold stress
9 < UTCI < 26	no thermal stress
26 < UTCI < 32	moderate heat stress
32 < UTCI < 38	strong heat stress
38 < UTCI < 46	very strong heat stress
46 < UTCI	extreme heat stress

Table 1: Universal Thermal Climate Index (UTCI) and corresponding thermal stress categories.

UTCI values are computed using the simplified polynomial form, using the exterior air temperature and assuming a constant vapor pressure $v_p = 0.67 \text{ kPa}$ in the cold range of exterior temperatures and $v_p = 1.67 \text{ kPa}$ in the hot range. Only sensible heating and cooling are included in this study, so the relative humidity increases with cooling and decreases with heating. The wind speed is set to $u = 0.5 \text{ m/s}$, the minimum for the UTCI model, which is equivalent to slight interior air movement. It is assumed that the radiant temperature is equal to the interior air temperature. The resulting limits are labeled along the secondary y-axis of Figure 8.

These UTCI thresholds provide reasonable limits for the quasi-interior and quasi-exterior setpoints T_2 and T_3 . They also support the use of fixed temperature setpoints outside the range of exterior temperatures $10^{\circ}\text{C} < T_{ext} < 30^{\circ}\text{C}$ covered by the ACM. The limits of T_1 are therefore set at the upper and lower values of the ACM band. T_2 is limited above and below by the UTCI thresholds for “very strong heat” and “slight cold stress” respectively, and T_3 by the thresholds for “extreme heat” and “moderate cold.” Until T_2 and T_3 exceed these thresholds, they are allowed to “float.” Essentially, they each follow a close-to-free-running temperature where there is no active conditioning in the space in question, but there is active conditioning in a more interior space. Heat is exchanged between adjoining spaces, offsetting the floating temperatures from their baseline free-running value and producing a temperature “cascade.”

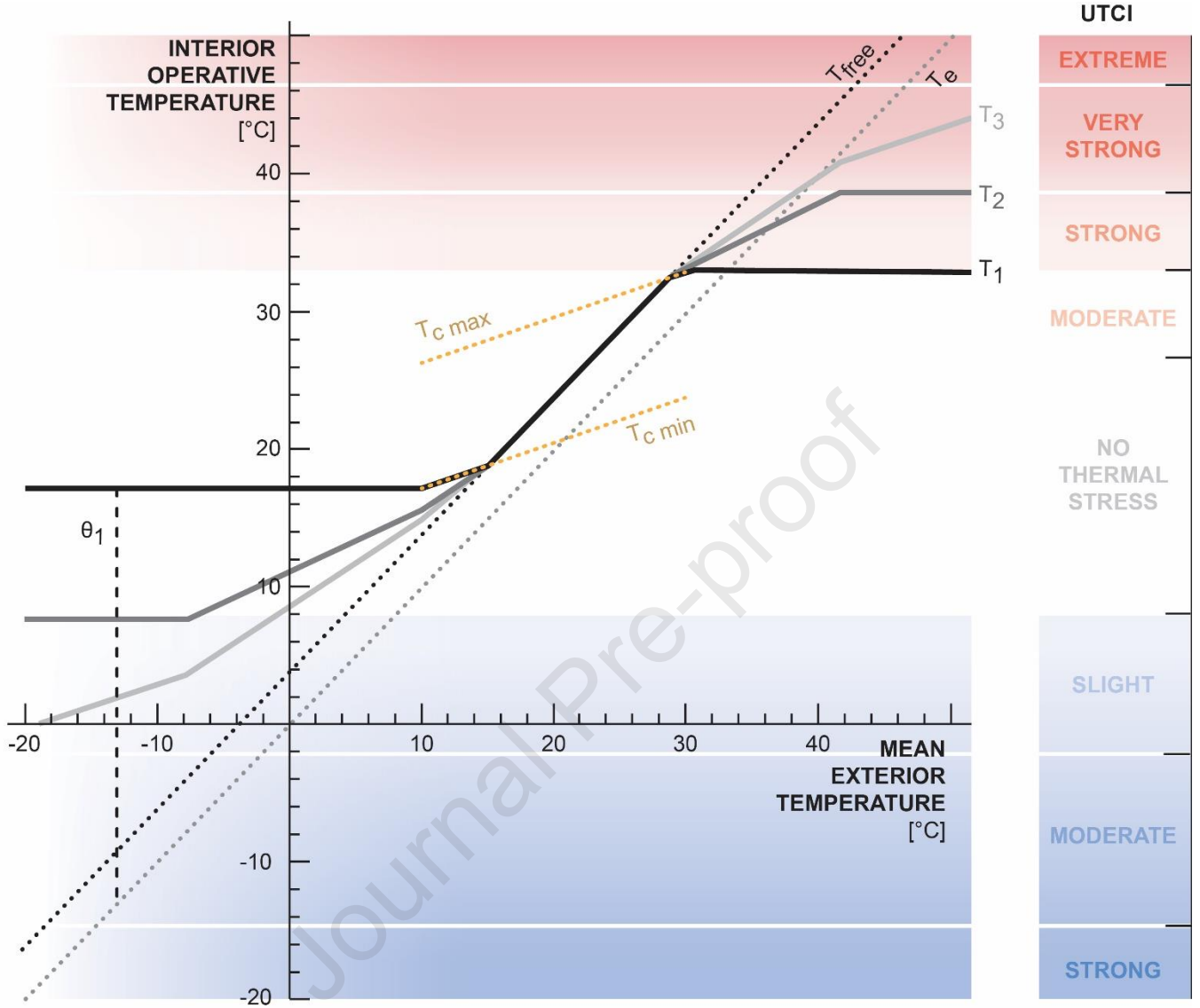


Figure 8: Setpoint temperature profiles for the interior (T_1), quasi-interior (T_2), and quasi-exterior spaces (T_3) as a function of the prevailing exterior temperature. Corresponding UTCI heat stress categories (Table 1) are indicated along the secondary y-axis. All spaces are free-running while T_{free} falls within the adaptive comfort band – otherwise, T_2 and T_3 are allowed to “float” in thermal cascade until they reach the prescribed thresholds for heat and cold stress.

The resulting profile for T_1 is a piecewise function,

$$T_1(x) = \begin{cases} UTCI_9, & x < 10 \\ T_{c \min}(x), & 10 < x < T_{free}(x) \cap T_{c \min}(x) \\ T_{free}(x), & T_{free}(x) \cap T_{c \min}(x) < x < T_{free}(x) \cap T_{c \max}(x) \\ T_{c \max}(x), & T_{free}(x) \cap T_{c \max}(x) < x < 30 \\ UTCI_{26}, & 30 < x \end{cases} \quad 39$$

where x has been substituted for T_{ext} for simplicity. $T_{free}(T_{ext}) \cap T_{c \min}(T_{ext})$ indicates the value of T_{ext} where the functions for the free-running temperature $T_{free}(T_{ext})$ and minimum comfort

temperature $T_{c\min}(T_{ext})$ intersect. All intersections are written following this notation. The term $UTCI_j$ represents the exterior temperature corresponding to a UTCI value of j , one of the threshold values between the different stress categories in Table 1.

The profiles for T_2 and T_3 are also described using piecewise functions:

$$T_2(x) = \begin{cases} UTCI_0, & x < T_{2f}(x) \cap UTCI_0 \\ T_{2f}(x), & T_{2f}(x) \cap UTCI_0 < x < T_{2f}(x) \cap UTCI_{32} \\ UTCI_{32}, & T_{2f}(x) \cap UTCI_{32} < x \end{cases} \quad 40$$

$$T_3(x) = \begin{cases} UTCI_{-13}, & x < T_{3f}(x) \cap UTCI_{-13} \\ T_{3f}(x), & T_{3f}(x) \cap UTCI_{-13} < x < T_{3f}(x) \cap UTCI_{38} \\ UTCI_{38}, & T_{3f}(x) \cap UTCI_{38} < x \end{cases} \quad 41$$

T_{2f} and T_{3f} indicate the “floating” temperatures of the corresponding spaces. These are found by solving the relevant temperature equations for a given nested scenario with $H_2 = H_3 = 0$.

There is a discrepancy between the temperature thresholds predicted by the UTCI and the ACM. This is not surprising; the two represent different approaches with different assumptions and are intended for different purposes. The ACM is correlated from a large dataset of empirical observations of interior thermal comfort in office buildings, while the UTCI is a biophysical heat balance intended to assess exterior thermal strain using a multi-node model of human thermoregulation. The two metrics treat personal adaptation via changing clothing levels differently; the ACM does not treat clothing as a variable, assuming occupants will adjust their clothing to their comfort level. In contrast, the UTCI explicitly defines clothing level as a function of the exterior temperature. The metabolic rate imposed by the simplified UTCI also corresponds to a constant walking pace rather than the sedentary or semi-sedentary activity assumed in the ACM. As a result, the UTCI might overestimate thermal stress in nested spaces during hot exterior temperatures, where higher activity levels would result in increased strain. However, it might be more appropriate for cold scenarios, where a higher metabolic rate works in favor of comfort.

The UTCI value with a lower metabolic rate can only be obtained by performing the full model calculation, which is beyond the scope of the current study. An alternative approach might be to use the UTCI for cool scenarios and use another heat index with a lower metabolic rate for hot ones. For example, the Physiologically Equivalent Temperature (PET) assumes a metabolic rate of 80 W/K/person , and has recently been used to study thermal comfort in nested courtyard spaces [12]. However, the PET assumes a fixed clothing level of 0.9 clo, which presents an additional discrepancy with the ACM and might result in similar overestimation of thermal strain in hot scenarios; the PET category for “moderate heat stress” begins at 29°C , which is not dissimilar from the UTCI polynomial value. Recognizing the limitations of the UTCI polynomial, the UTCI limit for “moderate heat stress” is only labeled on the secondary y axis and not included in the setpoint figure. At more extreme outdoor temperatures, the resulting setpoints may not provide *comfortable* spaces, but will at least provide safer conditions than outside. They may also reduce some thermal shock when transitioning from the exterior towards the interior. Most importantly, occupants may always take the adaptive behavior of moving towards the more comfortable interior.

4. Results and Discussion

4.1 Relative energy savings

All the components are now in hand to compute conditioning needs for a nested building compared to an equivalent single-space reference. Assuming internal heat gains are balanced (i.e., b_2 and b_3 are calculated using Eqs. 35-38) the equations are solved as follows:

1. Set the total envelope conductance UA , internal heat gains H_{int} , and flowrate Q .
2. Determine the proportion of the total UA allocated to each space i by setting a_i . For flow-divided configurations, determine the allocation of Q as well by setting c_i .
3. Compute the “balanced” values of the heat distribution parameters b_i in each space i using Eqs. 35-38.
4. Compute the conditioning needs by substituting the temperature profiles (Eqs 39-41) into Eqs. 2-19, using Eq. **Error! Reference source not found.** to compute the relative temperature θ_i from T_i for each space i .

Two cases are considered – one with a relatively low UA value of 5 W/K/person and one with a higher value of 50 W/K/person . The low UA value could represent a well-insulated envelope such as a traditional insulated wall assembly with a low U -value, or an envelope with a high U -value but a small surface area per occupant. The high UA value could represent a less-insulated envelope comprising mostly glazing or an uninsulated mass timber assembly. The values of the other parameters are identical for both cases (see Figure 10), with the exception of β and b_i , which specify the heat distribution. These are calculated from Eqs. 35-38. The overall internal heat gains are set to the value assumed by the UTCI summary polynomial ($H_{int} = 133.4 \text{ W/person}$) and the partition conductance is set to $UA_p = 10 \text{ W/K/person}$. A relatively high degree of nesting is employed, $\alpha = 0.75$, meaning that the sum of the envelope conductance values for the outer space(s) is three times that of the interior space. A double-space example of this with constant U for all envelope surfaces is given in Figure 11: a 4m tall building with an 8m x 8m footprint, which includes a 4m x 4m interior space nested in a corner.

The upper plots in Figure 9 show the sensible heating and cooling needs for both of these cases: the well-insulated envelope ($UA = 5 \text{ W/K/person}$) is on the left, and the less-insulated on the right ($UA = 50 \text{ W/K/person}$). Conditioning needs are given as a function of the mean exterior temperature, following the ACM. Curves are plotted for equivalent single-, double- and triple-space configurations, shown in black, gold, and blue, respectively. Flow-divided configurations are plotted as dotted lines and flow-connected as solid.

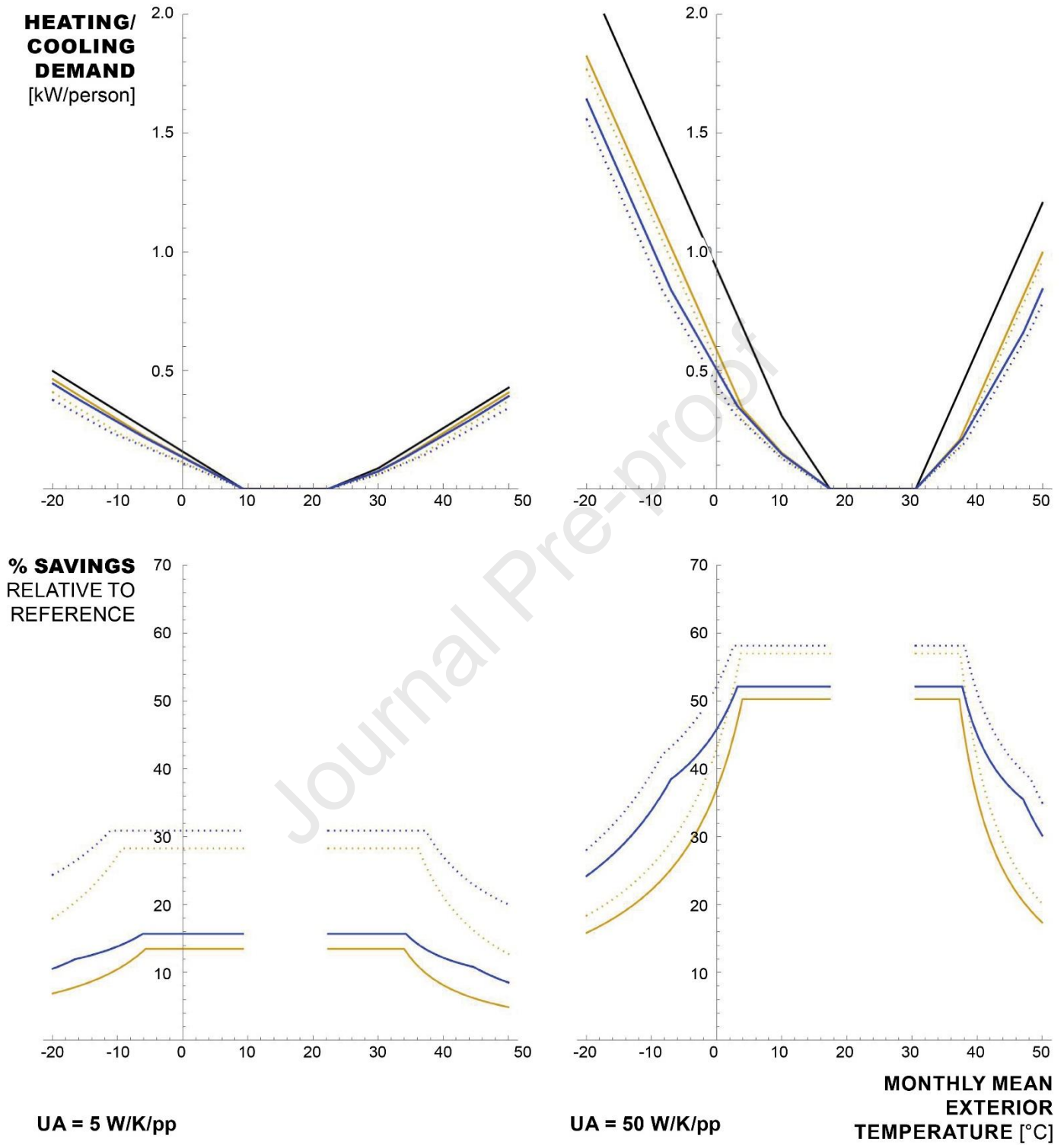


Figure 9: Heating/cooling demand (sensible) [Top] and relative savings [Bottom] for two-step (gold) and three-step (blue) cascades relative to the single-space reference case (black). Flow-connected configurations are plotted with solid lines, and flow-divided ones with dotted. Results are plotted for two cases: a highly-insulated envelope ($UA = 5 \text{ W/K/person}$, left) and a less-insulated one ($UA = 50 \text{ W/K/person}$, right). Parameter values are given in Figure 10.

It is immediately noted that sensible heating and cooling needs are zero in the free-running zone, where T_{free} passes through the ACM comfort band. Conditioning needs gradually increase as T_{ext} moves away from this range. Kinks appear in the curves at points where T_{ext} reaches thresholds defined by the ACM and UTCI (see Section 3.5) used to impose temperature setpoint limits.

The well-insulated case has a lower absolute energy demand than the less-insulated case, as less heat is lost to the exterior via the envelope. The difference between two- and three-step cascades is marginal for the well-insulated case, while it becomes significant for the less-insulated case. Envelope losses increase with larger UA values so that they represent a significant proportion of total losses. The addition of a third space means that a portion of these losses now occurs at the lower temperature T_3 , reducing overall demand.

For both cases, the flow-divided configurations require less energy than their flow-connected counterparts. In flow-divided mode, a portion of the ventilation losses occur at lower temperatures (T_2 and T_3) compared to the single-space reference, where all losses occur at the interior setpoint temperature T_1 .

These trends are reflected in the lower plots of Figure 9, which show the percentage savings achieved by nested configurations relative to the single-space reference. The well-insulated case achieves a maximum of ~15% savings for flow-connected configurations and ~30% for flow-divided ones. The less-insulated case, however, offers up to 50% savings for flow-divided cases and 60% for flow-divided ones. This suggests that nesting can significantly reduce energy demand relative to an equivalent single-space reference. Savings are maximized for milder exterior temperatures on either side of the free-running range where T_2 and T_3 are floating and the building is close-to-free-running. Savings from nesting decrease as exterior temperatures become more extreme, but three-step cascades begin to provide greater benefits relative to two-step in this range, since T_3 remains floating at more extreme values.

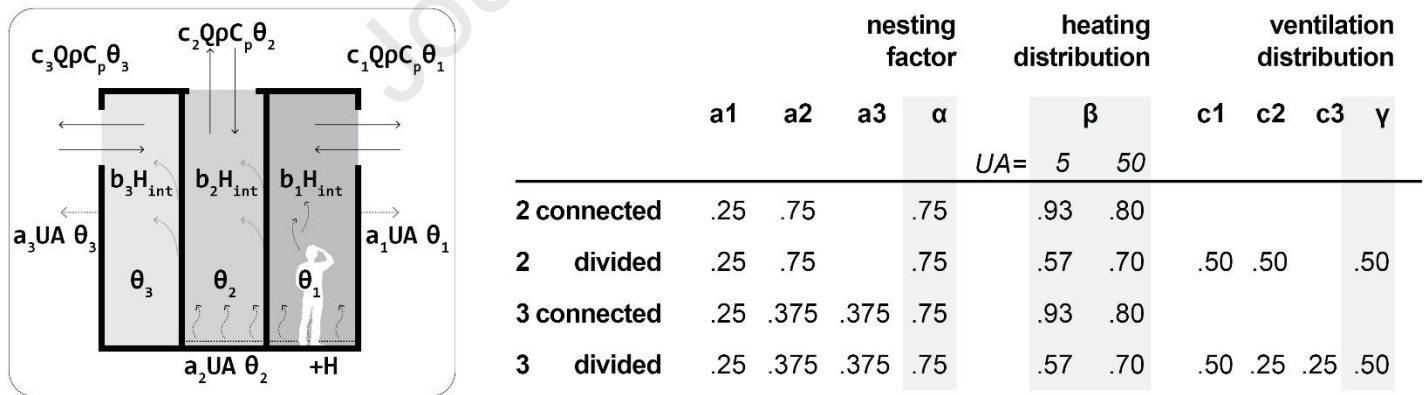


Figure 10: Design parameters for the cases evaluated in Figure 9. The building characteristics are $UA=5$ W/K/person for the well-insulated case and $UA=50$ W/K/person for the less-insulated case. $H_{int}=133.4$ W/person and $UA_p=10$ W/K/person for both cases. See Section 3.3 for parameter definitions.

The values of the parameters that were used to compute these plots are given in Figure 10. They specify the distribution of the total envelope conductance, internal heat gains and ventilation flowrate to each space, as explained in Section 3.3. The effect of varying these parameters is discussed in the following section.

4.2 Influence of design parameters

The nesting factor α describes the proportion of total envelope conductance assigned to the more exterior spaces: the quasi-interior space in two-step cascades and the quasi-interior *and* quasi-exterior spaces in three-step cascades. Figure 11 plots the maximum savings from nesting as a function of the total conductance UA for three values of α . As before, two-step cascading is plotted in gold, and three-step in blue, with flow-connected mode shown as solid lines and flow-divided as dotted lines.

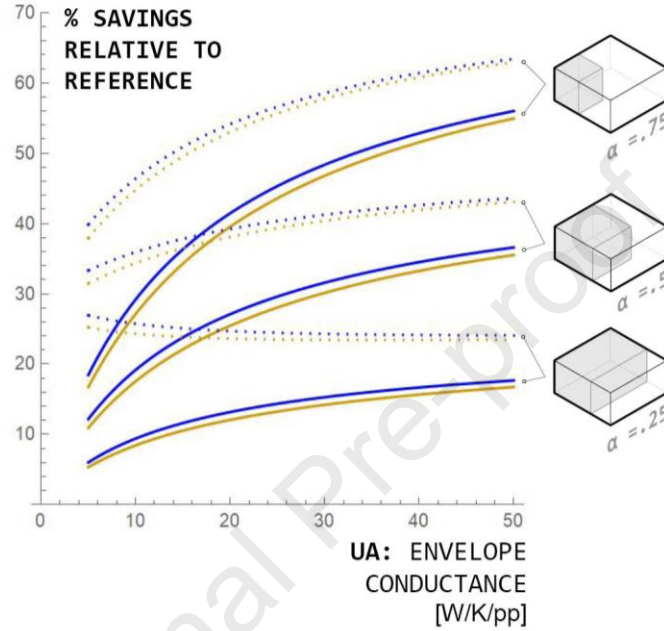


Figure 11: Maximum % savings for nested configurations relative to the reference case as a function of the total conductance UA and the nesting factor α . Curves are plotted for $\alpha = 0.25, 0.5$ and 0.75 . Values of the other parameters are given in Figure 10. Increasing UA and α both generate increased savings. [1.5 column fitting image]

As in Figure 9, results for three-step nesting are largely equivalent to two-step, with only a few percentage points of difference in savings between the two. Flow-divided configurations continue to provide greater savings overall than flow-connected ones. Most significantly, savings increase with both conductance and α , with the combination of high-conductance and high-nesting producing savings greater than 50%.

Varying the heating distribution β and ventilation distribution γ has less impact on savings than varying α does (not shown). Relative savings increase slightly with increasing γ , as a more significant proportion of the ventilation losses occur at the relatively lower temperatures in the quasi-interior and quasi-exterior spaces. Varying β (and thus moving away from the “balanced” free-running scenario discussed in Section 3.4) has opposite effects in heating and cooling. When β is below its balanced value (meaning that a greater proportion of the internal heat gains are in the interior space), the relative savings for cooling are greater than for heating, and the trend is inverted when β is above its balanced value. In the balanced scenario, the maximum relative savings are independent of the global internal heat gains H_{int} . However, increasing H_{int} does have the effect of increasing the free-running temperature and therefore shifting the plateau in maximum savings towards cooler exterior temperatures. Increasing the total range of the temperature cascade $|T_1 - T_2|$ and $|T_1 - T_3|$ also increases the potential savings of nesting relative to the reference case.

These results align with those of Pitts & ben Saleh [4] and may help to understand those of Kansara [5]. There is a clear correlation between the nesting factor for each layout analyzed in Pitts & ben Saleh and the relative energy savings that result from adjusted temperatures [4]. In Kansara's study, all the buildings where the mean operating temperature was raised have very low nesting factors (approximately 0.1) since the transition spaces are mainly hallways, stairways, and elevator lobbies [5]. The low nesting factor, combined with the small decrease in temperatures, limited the potential energy savings.

4.3 Model limitations

The present model assumes the flow and temperature fields have evolved to a steady-state. This implies scenarios where the transient effects of thermal mass, wind forcing, solar forcing, and changing internal heat loads are negligible. Internal radiant heat transfer and humidity were also not considered in the energy balance, while their influence in defining the comfort and heat stress thresholds was minimized. As such, the model should only be used in the preliminary stages of the design to size nested schemes according to a target temperature cascade. Figure 12 shows network diagrams for some of the nesting scenarios analyzed, as well as one which could be studied by adapting the present analysis. In this way, the model can support the preliminary design of nested schemes in new or retrofitted buildings and help define testable hypotheses for more detailed simulations or physical experiments.

When is it valid to neglect the transient effects of thermal mass? This assumption holds if the thermal mass inside the building is in thermal equilibrium with the indoor air, i.e. their temperature is always the same [39], [40]. Whether this is true depends on the rate at which temperature changes penetrate the mass, compared to the timescale ($t_1 \sim 1/\omega$) of forcing caused by diurnal fluctuations in the outside environment, whose frequency is given by ω [s^{-1}]. The rate at which temperature changes penetrate the mass depends on both the rate of convective heat transfer from the mass's surface h [$W/m^2/K$] to the indoor air (which occurs on a timescale $t_2 \sim l\rho_m C_m/h$) and on thermal diffusion through the mass (which occurs on a timescale $t_3 \sim l^2/\kappa_m$) [40]. l [m], ρ_m [m^3/s], C_m [$J/kg/K$] and κ_m [m^2/s] are the effective thickness, mass density, specific heat capacity and thermal diffusivity of the thermal mass, respectively. For steady conditions to be assumed, the ratios t_2/t_1 and t_3/t_1 must both be $\ll 1$. Otherwise, transient effects must be accounted for and the analysis must include convective heat transfer between the thermal mass and the indoor air [39], [40]. The flushing timescale $t_4 \sim V/Q$ required for the full volume of air V [m^3] in the room to be evacuated by the typical ventilation rate Q could also come into play in some situations [40]. However, the requirement $t_4/t_1 \ll 1$ is easily respected for most adequately ventilated buildings.

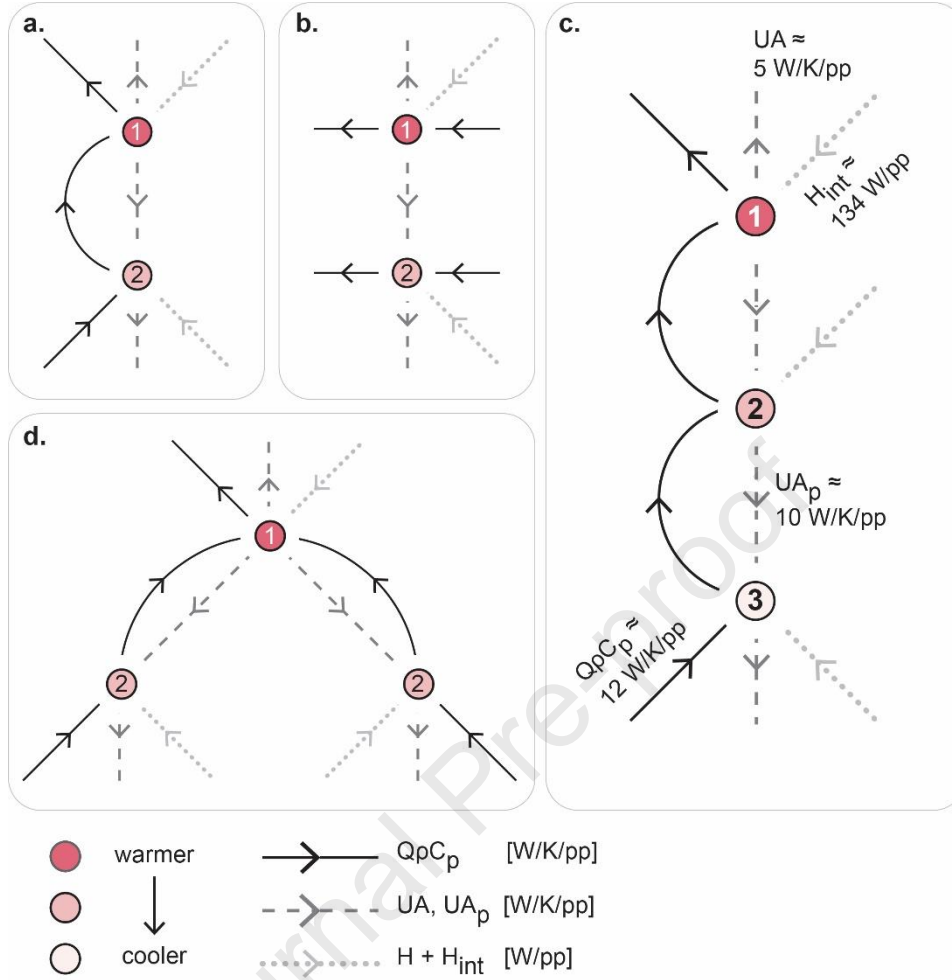


Figure 12: Network diagrams for nested configurations, showing the schedule of heat exchanges for (a) flow-connected and (b) flow-divided two-step buildings, (c) a three-step flow-connected building, and (d) a more complex nesting configuration with one interior space adjoining several distinct quasi-interior spaces. Relevant parameters (e.g. UA_p , UA , Q) could be adjusted to produce a desired temperature cascade or vice versa.

The transient effects of wind are more difficult to assess. When sufficiently strong relative to buoyancy effects, wind can significantly affect the flow rate in and out of a building and generate qualitative changes in the flow and temperature structure in the interior [24], [41]. Multiple flow regimes can exist for a given set of conditions with competing wind and buoyancy, and the flow may switch from one to the other, dependent on the history of the flow [42], [43]. Similarly, Chenvidyakarn and Woods indicate that multiple flow regimes are possible in flow-connected spaces in the absence of wind effects, although these elements may be controlled via vent sizing and the heating ratio between connected spaces [23]. The possibility of multiple regimes and potential impacts must be considered when analyzing the validity of the steady state assumption and in designing the control strategy.

Finally, the model assumes all spaces are well-mixed and of uniform temperature. This assumption is valid for a single space, so long as the heating or cooling is evenly distributed and not a point source [21], [22]. It is also valid for a two-step, flow-connected temperature cascade, so long as

the heating or cooling in both spaces is distributed evenly and horizontally along the floor or ceiling. [24]. In contrast, heated walls in a single ‘emptying filling box’ have been shown to produce a ‘peeling’ boundary layer, which tends to produce a stratified interior [28]–[30]. In a temperature cascade, heat recycling occurs through the partition walls. As such, it is not clear if mixing will occur in the more-exterior space if the partition wall is the principal source of heating.

4.4 Regional applicability

Thermal nesting can be understood as a strategy for increasing the feasibility of natural ventilation throughout the year and allowing occupants to adapt to future climate heating. The analysis in Figure 13 shows an estimate of current and future potential for natural ventilation—and hence thermal nesting—based on mean monthly temperatures predicted by the Coupled Model Intercomparison Project [44]. Each hexagon designates the land portion of an IPCC reference region – the shading of each segment indicates the months per year where natural ventilation is considered to be unviable by current adaptive comfort standards. According to this estimate, natural ventilation will become gradually less viable in hot regions such as Western Africa (labeled “WAF”), while it is likely to become increasingly useful in cooler parts of the world such as East North America (labeled “ENA”). Broadly speaking, there are regions where a warming climate will make natural ventilation more challenging, whereas in other regions, milder winters will present new opportunities for natural ventilation.

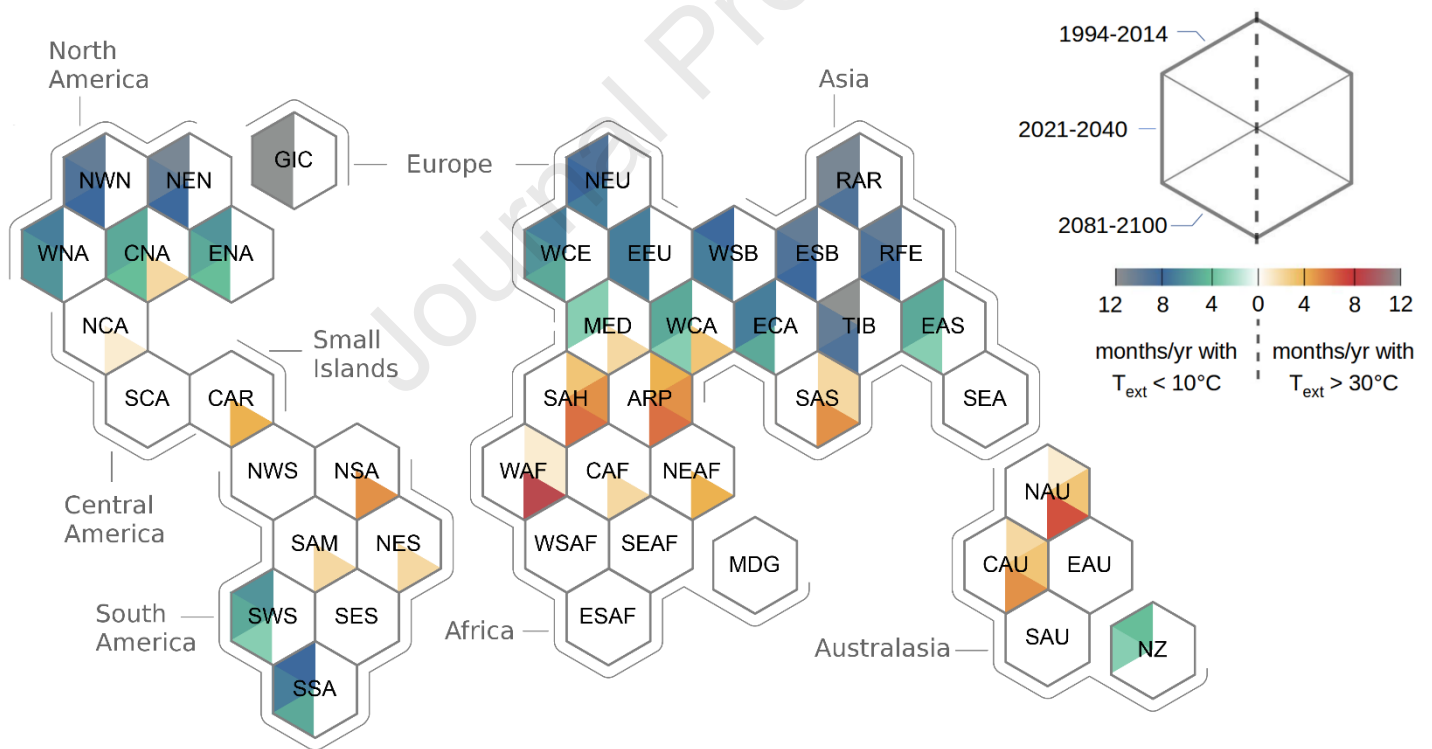


Figure 13: Evolving potential for natural ventilation as a function of time, by IPCC reference region [45], estimated from mean monthly temperatures from the recent past and predicted according to IPCC scenario SSP3 [46]. Each hexagon represents a region (excluding ocean and mountain areas). Shading indicates how many months of the year will be too cold (blue-green) or too hot (orange-red) for natural ventilation to provide thermal comfort according to adaptive comfort standard EN-16798-1:2019 [37]. Top, middle, and bottom segments represent conditions from the past (1994-2014), predictions for the short-term (2020-2040) and predictions for the medium-term (2081-2100) respectively. [2-column fitting image]

5. Conclusion

A temperature cascade refers to a series of spaces that are progressively warmer or cooler than the exterior. Thermal nesting refers to architectural configurations that produce temperature cascades. This study suggests how to design a thermally nested scheme that generates a natural temperature cascade, which occupants may adapt to with the change of seasons or during a heatwave. A steady-state model for an N-space temperature cascade ventilated by thermal buoyancy was developed. The thermal comfort and energy-saving limits were analyzed compared to a single space of uniform temperature, ventilated by natural mixing. Adaptive comfort and heat stress theory informed the choice of stepwise temperatures in the cascade. The most comfortable space in the cascade has supplementary heating or cooling, but only when necessary. The buoyancy ventilation occurs separately in each space or through each space in series. Heat recycling occurs through the partition walls.

The model was derived in several steps. First, the characteristics and energy balance for a single-space reference building were defined following Woods et al. [22]. The analysis was then adapted to two- and three-step temperature cascades, drawing on Chenvidyakarn and Woods [23]. The nesting factor, heating distribution, and ventilation distribution were defined to investigate how the distribution of envelope conductance, internal heat gains, and ventilation flow between spaces influences performance.

To define the stepwise temperatures in the temperature cascade, the Adaptive Comfort Model (EN 16798) was combined with the Universal Thermal Climate Index (UTCI) [37], [38]. Defining these temperature thresholds enabled calculation of the energy demand and relative savings of different nested building configurations compared to the reference building. The influence of design parameters on relative energy savings was then assessed.

Results show that thermal nesting has the potential to reduce sensible energy demand by more than 50% when the nesting factor is $\alpha \geq 0.75$. The relative energy savings increase with the degree of nesting, the benchmark envelope conductance, and the size of the temperature increments. Three-step cascades save slightly more energy than two-step cascades, because the envelope losses are smaller. Flow-divided configurations save more energy than flow-connected ones, because the ventilation heat loss is divided into smaller flows with smaller temperature increments.

Larger temperature steps between the spaces could increase savings even further, although occupant comfort and heat stress must be carefully evaluated beforehand. However, one difficulty is that temperature cascades are a hybrid of interior and exterior conditions, and there is no specific guidance for designing the thermal conditions in a temperature cascade. Further research in adaptive behaviours and dynamic thermal sensation in temperature cascades is required. Other areas of further research include incorporating transient effects into the model and investigating the fluid mechanics of temperature cascades, as outlined in section 4.3.

Generally speaking, designing adjacent rooms in a temperature cascade not only reduces energy demand in first-law terms but also creates opportunities for using and recycling low-grade heat (or cold) within the building. It is also true that working with low-grade heat requires large surface areas. Therefore, the use of heat-exchanging partition walls in a temperature cascade is a promising

area for further research. For instance, it is worth investigating how dehumidification could be incorporated downstream with radiant cooling upstream in hot, humid conditions.

Finally, it is worth highlighting one of the major challenges of climate change: sustainable cooling. The awful irony is that warmer temperatures lead to more air-conditioning, yet more air-conditioning leads to more GHG emissions [47]. Khosla et al. [48] identified five ‘levers’ needed to drive sustainable system change in the provision of cooling. Thermal nesting potentially engages three of these interconnected levers. These are: social interactions, such as the role of individual behaviours and lifestyles; technology innovation, including affordable passive and active cooling; and infrastructure design, such as space design that predetermines cooling demand.

CRediT author statement

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Highlights

- “Nested” buildings with cascading thermal zones could reduce temperature demand
- Analytical model shows energy savings of thermal nesting with natural ventilation
- Temperature increments set using adaptive comfort and heat stress theory
- Nesting could provide 50-60% savings relative to a building of uniform temperature
- Savings increase with wall conductance, temperature increments, and nesting factor

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.